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This equals the time to go from A to C .

For a perfectly rough tunnel, $v^2 = (dx/dt)^2 = (5g/7R)(a^2 - x^2) + v_1^2$.

$$\therefore t = \sqrt{7R} \int_0^a \frac{dx}{\sqrt{[7Rv_1^2 + 5g(a^2 - x^2)]}} = \sqrt{\frac{7R}{5g}} \sin^{-1} \frac{a\sqrt{5g}}{\sqrt{7Rv_1^2 + 5ga^2}}.$$

III. If the tunnel extends beyond the surface of the earth: then let SB be the tunnel. Above the earth's surface, acceleration varies inversely as the square of the distance from the center.

Let $CO=c$, $OQ=y$, $CQ=x$, $SC=h$. Then $f : g = R^2 : y^2$ or $f = gR^2/y^2 = gR^2/(c^2 + x^2)$. $f' : f = \sin \theta : 1$. $\therefore f' = f \sin \theta = fx/\sqrt{c^2 + x^2}$.

$$\therefore f' = \frac{gR^2 x}{\sqrt{(c^2 + x^2)^3}}. \quad \therefore \frac{d^2 x}{dt^2} + \frac{gR^2 x}{\sqrt{(c^2 + x^2)^3}} = 0.$$

$$\therefore \left(\frac{dx}{dt}\right)^2 = 2gR^2 \left[\frac{1}{\sqrt{(c^2 + x^2)}} - \frac{1}{\sqrt{(c^2 + h^2)}} \right] = v_1^2.$$

$$\therefore v_1 = R \sqrt{2g \left[\frac{1}{\sqrt{(a^2 + c^2)}} - \frac{1}{\sqrt{(h^2 + c^2)}} \right]} = R \sqrt{\frac{2g}{R} - \frac{2g}{\sqrt{(c^2 + h^2)}}}.$$

When the sphere reaches A ,

$$t = \frac{\sqrt[4]{(c^2 + h^2)}}{R\sqrt{(2g)}} \int_0^h \frac{\sqrt[4]{(c^2 + x^2)} dx}{\sqrt{\left[\frac{1}{\sqrt{(c^2 + h^2)}} - \frac{1}{\sqrt{(c^2 + x^2)}} \right]}}.$$

$$\therefore t = \frac{2(c^2 + h^2)^{\frac{5}{4}}}{R\sqrt{(2g)}} \int_{\phi_1}^{\frac{1}{2}\pi} \frac{\sin^4 \phi d\phi}{\sqrt{[(c^2 + h^2)\sin^4 \phi - c^2]}},$$

where $c^2 + x^2 = (c^2 + h^2)\sin^4 \phi$ and $\phi_1 = \sin^{-1} \sqrt{\frac{R^2}{c^2 + h^2}}$.

t = time to move from S to A . For a perfectly rough tunnel, $v_2 = \sqrt{\frac{5}{7}}v_1$; $t_1 = \sqrt{\frac{7}{5}}t$.

Also solved by S. Lefseletz.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

160. Proposed by H. S. VANDIVER, Bala, Pa.

Prove that the integer next above $(1 + \sqrt[3]{3})^{2n}$ is divisible by 2^{n+1} .

Proof by G. E. WAHLIN, Urbana, Ill.

$|1 - \sqrt{3}| < 1$. Hence (1) $0 < (1 - \sqrt{3})^{2n} < 1$.

But $(1 + \sqrt{3})^{2n} + (1 - \sqrt{3})^{2n} = E$, a rational integer, and since (1) is true, this is the integer next above $(1 + \sqrt{3})^{2n}$.

Since $\frac{(1 \pm \sqrt{3})^2}{2} = 2 \pm \sqrt{3}$, we have $\frac{(1 \pm \sqrt{3})^{2n}}{2^n} = (2 \pm \sqrt{3})^n = P \pm Q\sqrt{3}$, P and Q rational integers.

Then $\frac{(1 + \sqrt{3})^{2n}}{2^n} + \frac{(1 - \sqrt{3})^{2n}}{2^n} = \frac{E}{2^n} = 2P$. Therefore, $\frac{E}{2^{n+1}} = P$, and hence E is divisible by 2^{n+1} .

Also solved by J. Scheffer, G. B. M. Zerr, and V. M. Spunar.

PROBLEMS FOR SOLUTION.

ALGEBRA.

321. Proposed by C. C. BLAND, Attorney at Law, Rolla, Mo.

A corporation is capitalized for \$20,000. 125 shares of the par value of \$100 per share has been issued. A has 27 $\frac{19}{78}$ shares. B, C, D, E and F each have 19 $\frac{43}{78}$ shares. It is the wish of the corporation to cancel the certificates held by A, B, C, D, E, and F, and to issue new certificates to each of them in lieu of those now held by them, and to avoid the issuance of any certificate for a fraction of a share. How many shares should each receive, the whole not to exceed 200, at the same time maintaining the present interest of each in the corporation?

322. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

Take six consecutive prime numbers, as 53, 59, 61, 67, 71, and 73, and find the least whole number such that if it be divided by 59 the remainder will be 53, if it be divided by 67 the remainder will be 61, and if it be divided by 73 the remainder will be 71, and show that this least whole number and the succeeding consecutive whole numbers that will fulfill this condition as to divisions and remainders are in arithmetical progression; and also show whether or not this is a general law for n consecutive prime numbers; and if there be such a general law whether or not that general law will lead to a general law for the finding of prime numbers.

323. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

Show that the relation $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$ can hold for real numbers only when they are in proportion.

GEOMETRY.

348. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

Two parabolas and a rectangular hyperbola circumscribe a given quadrilateral. Find a relation between the squares of the latera recta of the parabolas and the squares of the perpendiculars from the center of the hyperbola to the axes of the parabolas.